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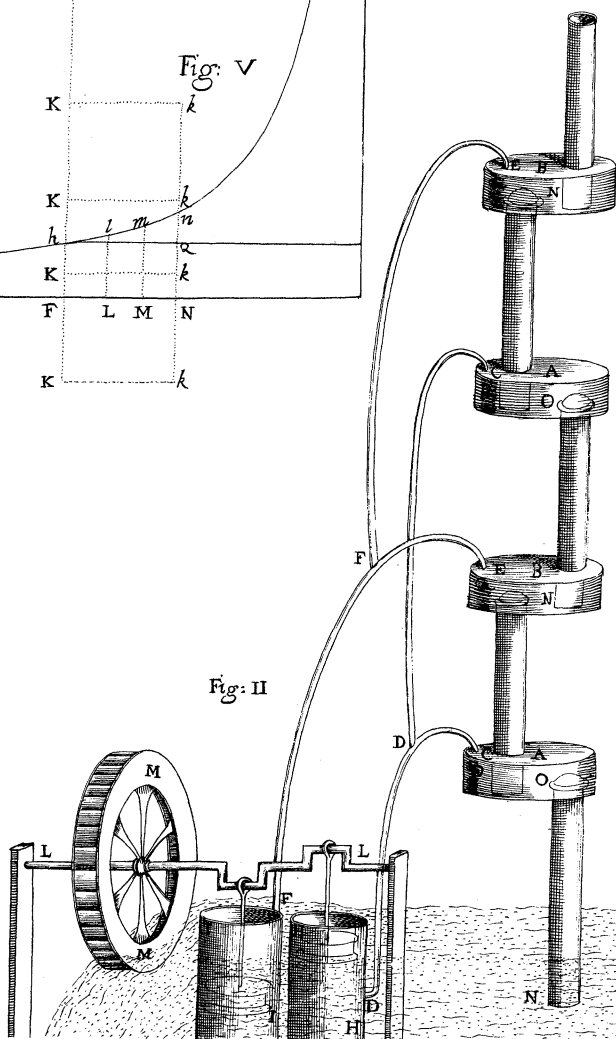
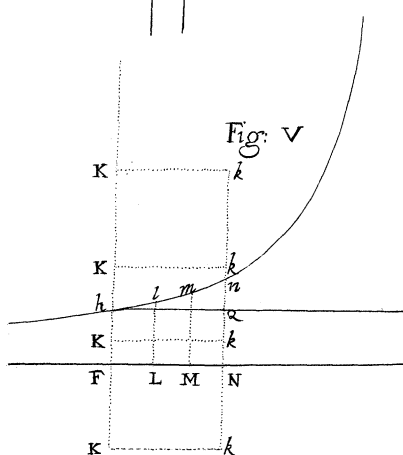
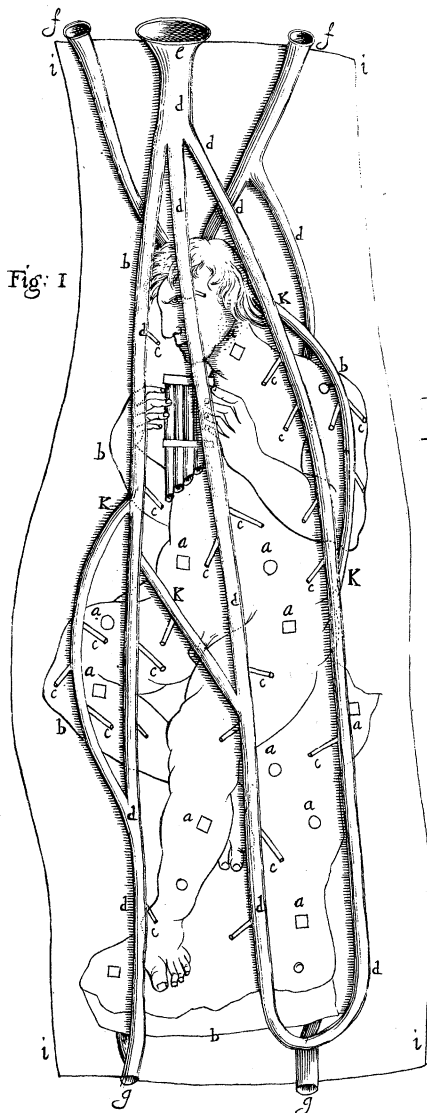
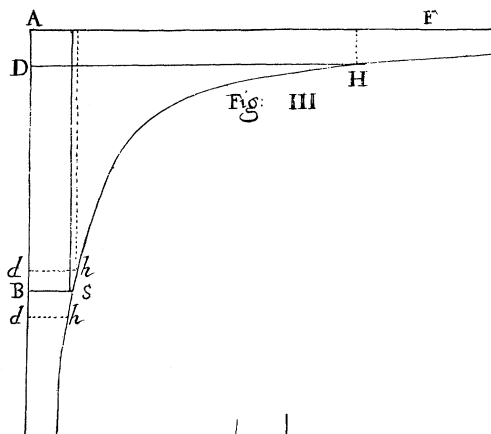
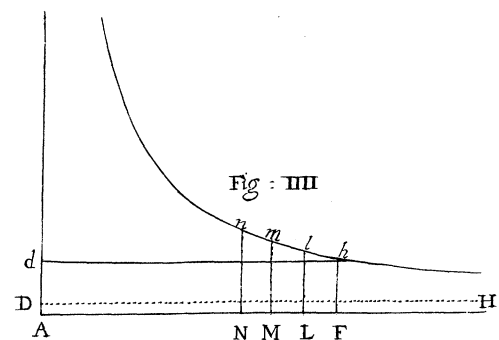
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A Discourse concerning the Measure of the Airs resistance to Bodies moved in it. By the Learned John Wallis S. T. D. & R. S. Soc.

1. **T**Hat the Air (and the like of any other *Medium*) doth considerably give resistance to Bodies moved in it, (and doth thereby abate their Celerity and Force :) is generally admitted. And Experience doth attest it : For otherwise, a Cannon Bullet projected Horizontally, should (supposing the Celerity and Force undiminished) strike as hard against a perpendicular Wall, erected at a great distance, as near at hand : which we find it doth not.

2. But at what Rate, or in what Proportion, such resistance is ; and (consequently, at what Rate the Celerity and Force is continually diminished) seems not to have been so well examined. Whence it is, that the Motion of a Project (secluding this Consideration) is commonly reputed to describe a Parabolick Line ; as arising from an Uniform or equal Celerity in the Line of Projection, and a Celerity uniformly accelerated in the Line of Descent : which two so compounded, do create a Parabola.

3. In order to the computation hereof ; I first premise this *Lemma*, (as the most rational that doth occur for my first footing,) That (supposing other things equal) the resistance is proportional to the Celerity. For in a double Celerity, there is to be removed (in the same time) twice as much Air, (which is a double Impediment) in a treble, thrice as much ; and so in other Proportions.

4. Suppose we then the Force impressed (and consequently the Celerity, if there were no resistance) as 1 ; the resistance as r . (which must be less than the Force, or else the Force would not prevail over the Impediment, to create a Motion.) And therefore the effective Force at a first Moment, is to be reputed as $1 - r$: That is, so much as

the Force impressed, is more than the Impediment or Resistance.

5. Be it as $1-r$ to 1 ; so 1 to m . (which m is therefore greater than 1 .)

6. And therefore the effective Force (and consequently the Celerity) as to a first Moment, is to be $\frac{1}{m}$ of what it would be, had there been no resistance.

7. This $\frac{1}{m}$ is also the remaining Force after such first Moment; and this remaining Force is (for the same Reason) to be proportionally abated as to a second Moment: that is we are to take $\frac{1}{m}$ thereof, that is $\frac{1}{m} \frac{1}{m}$ of the impressed Force. And for a third Moment (at equal distance of time) $\frac{1}{m} \frac{1}{m} \frac{1}{m}$; for a fourth $\frac{1}{m} \frac{1}{m} \frac{1}{m} \frac{1}{m}$; and so onward infinitely.

8. Because the length dispatched (in equal times) is proportional to the Celerities; the Lines of Motion (answering to those equal Times) are to be as $\frac{1}{m}, \frac{1}{m}^2, \frac{1}{m}^3, \frac{1}{m}^4$, &c. of what they would have been, in the same Times, had there been no resistance.

9. This therefore is a Geometrical Progression; and (because of m greater than 1) continually decreasing.

10. This decreasing Progression infinitely continued (determining in the same point of Rest, where the Motion is supposed to expire) is yet of a Finite Magnitude; and equal to $\frac{1}{m-1}$ of what it would have been in so much Time, if there had been no resistance. As is demonstrated in my Algebra, *Chap. 95. Prop. 8.* For (as I have elsewhere demonstrated) the Sum or Aggregate of a Geometrical Progression is $\frac{VR - A}{R - 1}$ (supposing V the greatest term, A the least, and R the common multiplier.) That is $\frac{VR}{R-1} - \frac{A}{R-1}$. Now in the present Case, (supposing the Progression infinitely continued) the least term A ,
be.

becomes infinitely small, or = 0. And consequently $\frac{A}{R-1}$ doth also vanish, and thereby the Aggregate becomes $= \frac{VR}{R-1}$. That is

(as will appear by dividing VR by $R-1$;) $V + \frac{V}{R} + \frac{V}{R^2} + \frac{V}{R^3} + \&c. = \frac{VR}{R-1}$;

(supposing the Progression to begin at $V=1$.) That is (dividing all by R , that so the Progression may begin at $\frac{V}{R} = \frac{1}{m}$;) $\frac{V}{R-1} = \frac{V}{R} + \frac{V}{R^2} + \frac{V}{R^3} + \&c.$ That is, in our present Case (because of $V=1, \& R=m$;) $\frac{1}{m} + \frac{1}{mm} + \frac{1}{m^3}$

$$\begin{array}{r} R-1)VR(V + \frac{V}{R}, \frac{V}{RR}, \&c. \\ \underline{VR-V} \\ +V \\ +V-\frac{V}{R} \\ \hline +\frac{V}{R} \\ +\frac{V}{R}-\frac{V}{RR} \\ \hline +\frac{V}{RR} \\ \&c. \end{array}$$

$\&c. = m-1$. That is, (putting $n = m-1$) $\frac{1}{n}$ of what it would have been if there had been no resistance.

11. This infinite Progression is fitly expressed by an ordinate in the exterior Hyperbola, parallel to one of the Asymptotes ; and the several Member of that, by the several Members of this, cut in continual Proportion. As is there demonstrated at Prop. 15. For let SH , (*vid.* Fig. III.) be an Hyperbola between the Asymptotes AB, AF : And let the ordinate DH (in the exterior Hyperbola, parallel to AF ;) represent the impressed force undiminished; or the Line to be described in such time, by a Celerity answerable to such undiminished force. And let BS (a like ordinate) be $\frac{1}{m}$ thereof ; which therefore, being less than DH

DH (as being equal to a Part of it) will be further than it from AF . In AB (which I put $= 1$) let Bd be such a Part thereof, as is BS of DH . Now becaufe (as is well known) all the inscribed Parellelograms, in the exterior Hyperbolá, $AS, AH, \&c.$ are equal; and therefore their

sides reciprocal: Therefore as $Ad = 1 - \frac{1}{m}$ (supposing Bd to be taken, from B toward A ,) to $AB = 1$, (or as $m - 1$ to m :) so is BS

$= \frac{1}{m} DH$, to dh , which $m - 1$) $1 (\frac{1}{m} + \frac{1}{mm} + \frac{1}{m^3} + \&c.$ is therefore equal to

$\frac{1}{m - 1}$ of DH ; that is (as will appear by dividing 1 , by $m - 1$,) to $\frac{1}{m} + \frac{1}{mm} + \frac{1}{m^3} \&c.$ of DH .

Or if Bd be taken beyond B ; then as $Ad = 1 + \frac{1}{m}$, to $AB = 1$, or as

$m + 1$ to m , so is $\frac{1}{m} DH$ to dh , which is therefore

equal to $\frac{1}{m + 1} DH$; that is (as will appear by like dividing of 1 by $m + 1$;) to $\frac{1}{m} - \frac{1}{mm} + \frac{1}{m^3} - \&c.$ of DH .

12. Let such ordinate dh , or (equal to it in the Asymptote) AF , be so divided in $L, M, N \&c.$ (by perpendiculars cutting the Hyperbola in $l, m, n, \&c.$) as that FL, LM, MN be as $\frac{1}{m}, \frac{1}{mm}, \frac{1}{m^3} \&c.$ That is, so continually decreasing, as that each antecedent be to its consequent, as 1 to $\frac{1}{m}$, or as m to 1 . See Fig. IV

13. This is done by taking $AF, AL, AN, \&c.$ in such proportion. For, of continual proportionals the differences are also continually proportional, and in the same pro-

$$\begin{aligned} & \frac{1 - \frac{1}{m}}{\frac{1}{m}} \\ & + \frac{\frac{1}{m}}{\frac{1}{m^2}} \\ & + \frac{\frac{1}{m} - \frac{1}{mm}}{\frac{1}{m^2}} \\ & + \frac{\frac{1}{m}}{\frac{1}{mm^2}} \\ & + \frac{\frac{1}{m} - \frac{1}{mm}}{\frac{1}{mm^2}} \\ & \&c. \quad + \frac{1}{mm^2} \end{aligned}$$

proportion. For let A, B, C, D , &c. be such proportionals; and their differences a, b, c , &c. That is $A - B = a$, $B - C = b$, $C - D = c$, &c.

Then, because A, B, C, D , &c. are in continual proport: That is $A. B :: B. C :: C. D :: \&c.$

And dividing $A - B. B :: B - C. C :: C - D. D :: \&c.$

That is $a. B :: b. C :: c. D :: \&c.$

And alternly $a. b. c. \&c. :: B. C. D. \&c. :: A. B. C. \&c.$

That is, in continual proportion as A to B , or as m to 1 .

14. This being done; the Hyperbolick spaces Fl , Lm , Mn ; &c. are equal. As is demonstrated by *Gregory San-Vincent*; and as such is commonly admitted.

15. So that Fl , Lm , Mn , &c. may fitly represent equal times, in which are dispatched unequal lengths, represented by FL , LM , MN , &c.

16. And because they are in number infinite (though equal to a finite Magnitude) the duration is infinite: And consequently the impressed force, and motion thence arising, never to be wholly extinguished (without some further impediment) but perpetually approaching to A , in the nature of Asymptotes.

17. The spaces Fl , Fm , Fn , &c. are therefore as Logarithms (in Arithmetical progression increasing) answering to the lines AF , AL , AM , &c.; or to FL , LM , MN , &c. in Geometrical progression decreasing,

18. Because FL , LM , MN , &c. are as $\frac{1}{m}$, $\frac{1}{m^2}$, $\frac{1}{m^3}$, &c. (infinitely) terminated at A ; therefore (by ¶ 10) their Aggregate FA or dh , is to DH , (so much length as would have been dispatched, in the same time, by such impressed force undiminished) as 1 to $m - 1 = n$.

19. If therefore we take, as 1 to n , so AF to DH ; this will represent the length to be dispatched, in the same time, by such undiminished force.

20. And if such DH be supposed to be divided into equal parts innumerable (and therefore infinitely small;) these answer to those (as many) parts unequal in FA , or hd .

21. But, what is the proportion of r to 1 , or (which depends on it) of $1 - r$ to 1 , or 1 to m ; remains to be inquired by experiment.

22. If the progreſſion be not infinitely continued; but end (ſuppoſe) at N , and its leaſt term be $A = MN$: then, out of $\frac{V}{R-1} = \frac{1}{m} + \frac{1}{m^2} + \frac{1}{m^3}$, &c. is to be ſubducted

$\frac{A}{R-1}$ (as at ¶ 10) that is (as by diviſion will appear)

$\frac{A}{R} + \frac{A}{R^2} + \frac{A}{R^3}$ &c. That is (in our preſent caſe) $\frac{a}{m} +$

$\frac{a}{m^2} + \frac{a}{m^3}$ &c. And ſo the Aggregate will be $\frac{1-a}{m} + \frac{1-a}{m^2}$

$+ \frac{1-a}{m^3}$ &c. $= \frac{1-a}{n}$.

And thus as to the line of Projection, in which (ſecluding the reſiſtance) the motion is reputed uniform; diſpatching equal lengths in equal times. Conſider we next the line of Deſcent.

23. In the Deſcent of Heavy Bodies, it is ſuppoſed, that to each moment of time, there is ſuperadded a new Impulſe of Gravity to what was before: And each of theſe, ſecluding the conſideration of the Airs reſiſtance, to proceed equally (from their ſeveral beginnings) through the ſucceeding moments. As (in the erect lines)

1 1 1 1 &c. 1 1 1 &c, 1 1 &c. 1 &c. and ſo continually as in the line of of Projection.

1

1 1

24. Hence ariſeth (in the tranſverſe lines)

1 1 1

for the firſt moment , for the ſecond 1 + 1,

1 1 1 1

for the third 1 + 1 + 1, and ſo forth, in A-

&c.

riſtical progreſſion: As are the Ordinates in a Triangle, at equal diſtance.

25. And ſuch are the continual increments of the Diameter, or of the ordinates in the exterior Parabola, anſwering to the interior Ordinates, or Segments of the Tangent

gent, equally increasing. As is known, and commonly admitted.

26. If we take-in the consideration of the Airs resistance ; we are then for each of these equal progreffions, to substitute a decreasing progreffion Geometrical ; in like manner (and for the same reasons) as in the line of Projection.

27. Hence ariseth, for the first moment m ; for the second $m + m^2$; for the third $m + m^2 + m^3$ &c. And such is therefore the Descent of a heavy Body falling by its own weight. The several impulses of Gravity being supposed equal.

28. That is (in the figure of ¶ 12) as FL, FM, FN , &c, in the line of Descent, answering to FL, LM, MN , &c. in the line of Projection.

29. But though the Progreffions for the line of Projection, are like to each of those many in the line of Descent : it is not to be thence inferred, that therefore $\frac{1}{m}$ in the one, is equal to $\frac{1}{m}$ in the other : But in the line of Projection (suppose) $\frac{1}{m} f$ (such a part of the force impressed, and a celerity answerable :) in the line of Descent, $\frac{1}{m} g$ (such a part of the Impulse of Gravity.)

30. Those for the line of Descent (of the same Body) are all equal, each to other : Because g (the new Impulse of Gravity) in each moment is supposed to be the same.

31. But what is the proportion of f to g (that of the force impressed, to the Impulse of Gravity in each Body) remains to be enquired by Experiment.

32. This proportion being found as to one known force ; the same is thence known as to any other force

M m

(who's

(who's proportion to this is given) in the same uniform *Medium*.

33. And this being known as to one *Medium* ; the same is thence known as to any other *Medium*, the proportion of who's resistance to that of this is known.

34. If a heavy body be projected downward in a perpendicular line ; it descends therefore at the rate $\frac{1}{m}$, $\frac{1}{mm}$, $\frac{1}{m^3}$, &c. of f (the impressed force) increased by $\frac{1}{m}$, $\frac{1}{m} + \frac{1}{m^2}$, $\frac{1}{m} + \frac{1}{m^2} + \frac{1}{m^3}$ &c. of g the impulse of Gravity : (by ¶ 7. & ¶ 27.) Because both forces are here united.

35. If in a perpendicular projection upwards ; it ascends in the rate of the former, abated by that of the latter. Because here the impulse of Gravity is contrary to the force impressed.

36. When therefore this latter (continually increasing) becomes equal to that former (continually decreasing) it then ceases to ascend ; and doth thenceforth descend at the rate wherein the latter continually exceeds the former.

37. In an Horizontal or Oblique projection : If to a Tangent who's increments are as FL , LM , MN , &c ; that is as $\frac{1}{m}f$, &c. be fitted Ordinates (at a given angle) who's increments are as FL , FM , FN , &c. that is as $\frac{1}{m}g$, &c : The Curve answering to the compound of these Motions, is that wherein the Project is to move.

38. This Curve (being hitherto without a name) may be called *Linea Projectorum* ; the line of Projects, or things projected ; which resembles a Parabola deformed.

39. The Celerity and Tendency, as to each point of this line, is determined by a Tangent at that Point.

40. And that against which it makes the greatest stroke
or

or percussion, is that which (at that point) is at right angles to that Tangent.

41. If the Projection (at ¶ 27.) be not infinitely continued, but terminate (suppose) at N , so that the last term in the first Column or Series erect be a ; and consequently in the second, ma ; in the third, mma , &c. (each Series having one term fewer than that before it :) then (for the same reasons as at ¶ 22.) the Aggregates of the several Columns (or erect Series) will be $\frac{1-a}{n}$, $\frac{1-ma}{n}$,

$\frac{1-mma}{n}$, and so forth, till (the multiple of a becoming $=1$) the progression expire.

42. Now all the abatements here, a , ma , mma , &c. are the same with the terms of the first Column taken backward. For a is the last, ma the next before it; and so of the rest.

43. And the Aggregate of all the Numerators is so many times 1 as is the number of terms (suppose t ,) wanting the first Column; that is $t - \frac{1-a}{n}$, or $\frac{nt-1+a}{n}$; & this again divided by the common denominator n , becomes $\frac{nt-1+a}{nn}$. And therefore $\frac{nt-1+a}{nn} g$, is the line of descent by its own Gravity.

44. If therefore this be added to a projecting force downward in a perpendicular; or subducted from such projecting force upward; that is, to or from $\frac{1-a}{n} f$: The Descent in the first case will be $\frac{1-a}{n} f + \frac{nt-1+a}{nn} g$; and the Ascent in the other case $\frac{1-a}{n} f - \frac{nt-1+a}{nn} g$. And in this latter case, when the ablative part becomes equal to the positive part, the Ascent is at the highest: and
M m thence-

thenceforth (the ablative part exceeding the positive) it will descend.

45. In an Horizontal or Oblique projection ; having taken $\frac{1-a}{n}f$ in the line of Projection, and thence (at the

Angle given) $\frac{nt-1+a}{nn}g$ in the line of Descent ; the point in the Curve answering to these, is the place of the Project answering to that moment.

46. I am aware of some Objections to be made, whether to some points of the Process, or to some of the Suppositions. But I saw not well how to wave it, without making the Computation much more perplexed. And in a matter so nice, and which must depend upon Physical Observations, it will be hard to attain such accuracy as not to stand in need of some allowances.

47. Somewhat might have been further added to direct the Experiments suggested at ¶ 21. and 31. But that may be done at leisure, after deliberation had, which way to attempt the Experiment.

48. The like is to be said of the different resistance which different Bodies may meet with in the same *Medium*, according to their different Gravities (extensively or intensively considered) and their different figures and Positions in Motion. Whereof we have hitherto taken no account ; but supposed them, as to all these, to be alike and equal.

Post-script.

49. The computation in ¶ 41, 42, 43, may (if that be also desired) be thus represented by Lines and Spaces. The Ablatives $a, m a, m m a$, &c. (being the same with the first Column taken backward) are fitly represented by the segments of NF (beginning at N) in Figure IV. and V. and therefore by Parallelograms on these Bases, assuming the common height of Fb , or NQ : the Aggregate of which
is

is Nh , or FQ . And, so many times 1, by so many equal spaces, on the same Bases, between the same Parallels terminated at the Hyperbola: The Aggregate of which is $hFNQn$. From whence if we subduct the Aggregate of Ablatives FQ ; the remaining trilinear hQn , represents the Descent.

50. If to this of Gravity, be joyned a projecting Force; which is to the impulse of Gravity as hK to hF (be it greater, less, or equal) taken in the same line: the same parallels determine proportional Parallelograms, whose Aggregate is KQ .

51. And therefore if this be a Perpendicular Projection downwards; then $hKkn$ (the summe of this with the former) represents the Descent.

52. If it be a Perpendicular upwards; then the difference of these two represents the Motion: which so long as KQ is the greater, is Ascendent: but Descendent when hQn becomes greater: and it is then at the highest when they be equal.

53. If the Projection be not in the same Perpendicular, (but Horizontal, or Oblique) then KQ represents the Tangent of the Curve; and hQn the Ordinates to that Tangent, at the given Angle.

54. But the Computation before given I take to be of better use than this representation in Figure. Because in such Mathematical enquiries, I choose to separate (as much as may be) what purely concerns Proportions; and consider it abstractly from lines or other matter where-with it is incumbered.

As to the question proposed; whether the resistance of the *Medium* do not always take off such a proportional part of the force moving through it, as is the Specifick Gravity of the *Medium* to that of the Body moved in it: (for, if so, it will save us the trouble of Observation.)

I think this can by no means be admitted. For there be many other things of consideration herein, beside the In-

ten-

tenſive Gravity (or, as ſome call it, the Specifick Gravity) of the *Medium*.

A viſcous *Medium* ſhall more reſiſt, than one more fluid, though of like Intenſive Gravity.

And a ſharp Arrow ſhall bore his way more eaſily through the *Medium*, than a blunt headed Bolt, though of equal weight, and like intenſive Gravity.

And the ſame Pyramide with the Point, than with the Baſe forward.

And many other like varieties, intended in my ¶ 48.

But this I think may be admitted, namely, That different *Mediums*, equally liquid, (and other circumſtances alike,) do in ſuch proportion reſiſt, as is their Intenſive Gravity. Becauſe there is, in ſuch Proportion, a heavier object to be removed, by the ſame Force. Which is one of the things to which ¶ 33. refers.

And again: The heavier Project once in motion, (being equally ſwift, and all other circumſtances alike) moves through the ſame *Medium* in ſuch proportion more ſtrongly, as is its Intenſive Gravity. For now the Force is in ſuch proportion greater, for the removal of the ſame reſiſtance. And this part of what my ¶ 32. inſinuates.

But where there is a complication of theſe conſiderations one with another, and with many other circumſtances whereof each is ſeverally to be conſidered: there muſt be reſpect had to all of them.